Int. J. Heat Mass Transfer. Vol. 32, No. 10, pp. 1979–1987, 1989
Printed in Great Britain

0017-9310/89 $3.00+0.00

Shock wave formation around a moving heat source in a solid with finite speed of heat propagation

DA YU TZOU
Department of Mechanical Engineering, University of New Mexico, Albuquerque, NM 87131, U.S.A.

(*Received* 13 *October* 1988 *and in final form* 22 *February* 1989)

Abstract—The thermal field around a moving heat source in a solid with finite speed of heat propagation is studied analytically in this work. A thermal Mach number M defined as the ratio between the speed of the moving heat source and that of the heat propagation in the solid is introduced in the analysis. The resulting energy equation is found to be elliptic, parabolic, and hyperbolic in the subsonic ($M < 1$), transonic ($M = 1$), and supersonic ($M > 1$) ranges, respectively. Thermal shock wave is shown to exist in the physical domain as the speed of the moving heat source is equal to or faster than that of the heat propagation, and the thermal shock angle is obtained analytically as $\sin^{-1}(1/M)$ for $M \geqslant 1$. In the numerical examples, the evolution of the temperature and the heat flux distributions in the heat affected zone is present as a function of the thermal Mach number and a swinging phenomenon for the thermal field in transition is discussed.

1. INTRODUCTION

THE MERIT of the thermal wave model (or called hyperbolic theory of heat conduction) lies in its description on the thermal disturbance propagating with a finite speed in the solid. In applying such a theory, a general feature is that a distinct thermal wave front exists in the physical domain which separates, in the absence of reflecting waves, the heat affected zone from the thermally undisturbed zone. This is a different situation from that in the thermal diffusion model where the heat propagation speed is assumed to be infinite such that the existence of a thermal agent (including the boundary/initial conditions or heat sources) at infinity can be detected right after its application.

The mathematical form of the hyperbolic thermal wave equation was first proposed in ref. [1]. Under the absence of the heat source, it can be written as

$$\nabla^2 T = (1/\alpha)T_t + (1/C^2)T_{tt} \tag{1}$$

where C and α are the finite speed of heat propagation and the thermal diffusivity of the solid, respectively. The importance of considering the wave nature in a heat transport process is evaluated by comparing the two coefficients $1/\alpha$ and $1/C^2$ on the right-hand side of this equation. The criterion obtained in this manner is a function of intrinsic properties of the solid medium. Chester [2] established a critical frequency C^2/α (which is inversely proportional to the relaxation time) for thermal fluctuation above which heat transport proceeds by wave propagation rather than by diffusion. Later, Weymann [3] further identified the analogy between random walk and diffusion, and extended the study to the problems of mass diffusion and viscous shear motion.

The wave phenomena represented by equation (1) have been investigated from various physical points of view. In summary, it includes the modification of thermodynamics for an irreversible process [4] and thermodynamics with fading memory [5], incorporation of the kinetic theory of molecules [6, 7], and relativistic considerations for heat transport process [8, 9]. The limitations of the thermal diffusion model are also discussed in detail in these works.

The mathematical structure of the governing equations for the hyperbolic theory of heat conduction was also investigated [10]. It has been shown that a general flux formulation is more convenient to use than the standard temperature formulation for analysis in situations involving specified heat flux conditions. In the one-dimensional case without heat source, unlike the thermal diffusion model, the field equations governing the heat flux and the temperature in the thermal wave model have the same mathematical form.

Generally speaking, the thermal diffusion model is not appropriate to use under the situations involving very short time response, large temperature gradients, or low temperature circumstance. In engineering applications, these conditions have been examined via various initial and boundary value problems [11–14]. There is a class of problems dealing with impingement of a high intensity energy source on the surface of a structure which needs special consideration of the thermal wave model [15]. For this kind of problem, an extremely high temperature level is built up in the material continua adjacent to the energy source in a very short time duration. Due to localization of the energy source, such a response also develops large temperature gradients in the neighborhood. Consideration of the wave model in the heat transport process becomes even more important if some irreversible physical processes, such as crack or void initiation in the solid, occurs in the radiant duration. Under the application of an energy source with

NOMENCLATURE

c	parameter used in the thermal wave equation, $v/2\alpha$ [m^{-1}]
C	speed of heat propagation in the solid [$m\,s^{-1}$]
f	spatial distribution in the transformation on the dependent variables
G	Green's function for the transformed equation of f
k	thermal conductivity [$W\,m^{-1}\,K^{-1}$]
M	thermal Mach number, v/C
q	heat flux vector [$W\,m^{-2}$]
Q	intensity of the heat source [$W\,m^{-1}$]
r	transformation function for the independent variables [m]
R	radial distance from the heat source [m]
S	general heat source term [$W\,m^{-1}$]
t	physical time [s]
T	temperature [K]
v	speed of the moving heat source [$m\,s^{-1}$].

Greek symbols

α	thermal diffusivity [$m^2\,s^{-1}$]
δ	Dirac delta function
θ	angle in the polar coordinate system [deg]
ρ	mass density [$kg\,m^{-3}$]
ϕ	angle measured from negative ξ_1-axis [deg].

Mathematical symbols, superscripts, and subscripts

∇	gradient operator [m^{-1}]
∇^2	Laplacian operator [m^{-2}]
$(\ )_{,i}$	$\partial/\partial\xi_i$, $i = 1, 2$
$(\ )_{,t}$	$\partial/\partial t$
$(\bar{\ })$	dimensionless quantity
C_p	heat capacity [$kJ\,kg^{-1}\,K^{-1}$]
f_g	Green's function used in the subsonic and supersonic cases
I_n	modified Bessel function of the first kind of order n
K_n	modified Bessel function of the second kind of order n
q_1, q_2	heat flux components in the ξ_1- and ξ_2-directions [$W\,m^{-2}$]
r^0, r_η	radial position of the heat source [m]
T_g	Green's function for the transonic case
x_i	stationary coordinate system, $i = 1, 2$ [m]
θ_M	thermal shock angle [deg]
ξ_i	moving coordinates with the heat source, $i = 1, 2$ [m].

sufficiently high intensity, the local defects could be initiated in a much shorter time interval than that required for the diffusion behavior to be retrieved, and the orientation of crack initiation, for example, must be predicted according to the thermal wave model.

It is the intention of the present work to make a thorough study on the hyperbolic thermal field around a moving heat source. A thermal Mach number M defined as the ratio between the speed of the moving heat source and that of the heat propagation in the solid is introduced in the formulation. The temperature distributions are obtained in closed form solutions in various ranges of M. In the subsonic range with $M < 1$, the emphasis is placed on the comparison of the temperature and the heat flux distributions predicted by the thermal diffusion and the thermal wave models. While at the transonic and in the supersonic ranges with $M = 1$ and $M > 1$, respectively, we will show the formation of thermal shock waves and the evolution of the thermally undisturbed zone in the physical domain. The thermal shock angle is obtained as $\sin^{-1}(1/M)$ for $M \geqslant 1$. These are the salient features of the hyperbolic wave model which cannot be depicted by the diffusion model. Also, in transition of the thermal Mach number from the subsonic to the supersonic ranges, a swinging phenomenon involving the variations of the temperature as well as the heat flux components is obtained and discussed in detail.

2. FORMULATION OF THE PROBLEM

As shown in Fig. 1, consider a point heat source with intensity Q moving at a constant speed v along the x_1-axis. The dimensions of the solid are assumed to be large such that the edge effects on the local thermal field around the heat source can be neglected. It is well known that, however, the edge effects introduced through boundary conditions only influence

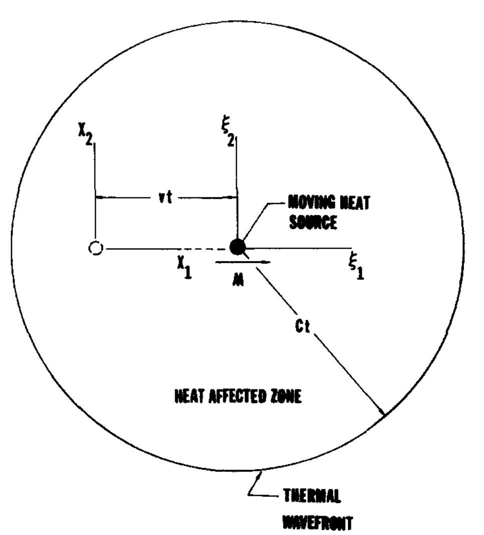

FIG. 1. Thermal waves emanating from a moving heat source and the coordinate system.

the intensity of the local field solutions while the characteristics of the solutions are not affected. The energy and the constitutive equations for the thermal wave propagating in the solid with a finite speed C can be written as

$$-\nabla\cdot q+S = \rho C_p T_t \quad (2)$$

$$(\alpha/C^2)q_t+q = -k\nabla T. \quad (3)$$

By assuming all the physical quantities being constant, we may eliminate the heat flux vector q from these equations and obtain a single equation governing the temperature T

$$\alpha\nabla^2 T+(1/\rho C_p)[S+(\alpha/C^2)S_t] = (\alpha/C^2)_{tt}+T_t. \quad (4)$$

Note that the apparent heat source term differs from the real heat source applied to the solid by a term of $(\alpha/\rho C_p C^2)S_t$.

For a point heat source with its position simulated by Dirac delta functions

$$S(x_1,x_2,t) = Q\delta(x_1-vt)\delta(x_2). \quad (5)$$

Equation (4) can be reduced to the form of

$$\alpha\nabla^2 T-(\alpha/C^2)T_{tt}-T_t = -(Q/\rho C_p)[\delta(x_1-vt) + (\alpha/C^2)\delta_t(x_1-vt)]\delta(x_2). \quad (6)$$

Since the heat source is assumed to be scanning on the solid at a constant speed, Galilei's transformation

$$x_1 = \xi_1+vt$$
$$x_2 = \xi_2 \quad (7)$$

can be applied, and the equation governing the quasi-stationary temperature field observed from the moving coordinate system ξ_i ($i = 1,2$) can be derived by substituting equation (7) into equation (6)

$$\alpha[(1-M^2)T_{,11}+T_{,22}]+2c\alpha T_{,1} = -(Q/\rho C_p)\{\delta(\xi_1)\delta(\xi_2)-(M^2/2c)\delta_{,1}(\xi_1)\delta(\xi_2)\} \quad (8)$$

where M is the thermal Mach number of the moving heat source with reference to the speed of heat propagation in the solid. Mathematically, $M = v/C$. The parameter c in equation (8) is defined as $v/2\alpha$.

The advantage of using equation (8) for the present study is quite obvious. The effects of the speed of the moving heat source and that of the heat propagation are concentrated on a single parameter M. The characteristics of the solution for equation (8) depend on the thermal Mach number. In the subsonic range with $M < 1$, the equation is elliptic. While at the transonic ($M = 1$) and in the supersonic ranges ($M > 1$), the equation is transited to be parabolic and hyperbolic, respectively. This is a situation similar to the equation governing the aerodynamic velocity potential in a mixed type of flow field [18]. The heat flux vector q in the ξ_i-coordinates is related to the temperature T by the following equation:

$$(M^2/2c)q_{,1}-q = k\nabla T. \quad (9)$$

As the speed of heat propagation in the solid approaches infinity, i.e. M approaches zero, equations (8) and (9) are reduced to those defined in the diffusion model. The solutions for T and q satisfying equations (8) and (9) in various ranges of M are nontrivial. Due to the presence of extra terms containing $T_{,1}$ and $\delta_{,1}$ in equation (8), the algorithm involved in the present analysis is more complicated than that in the classical theory of aerodynamics. In the following section, the energy equation (8) will be first transformed into a form for which the Green's function can be found. Both the dependent (T) and the independent (ξ_i) variables are involved in the transformation which varies in the three ranges of M. Closed-form solutions for T in the subsonic, transonic, and supersonic ranges are then followed by direct integrations. It is more explicit in this manner to display the influences of the finite speed of heat propagation on the thermal field around the heat source.

3. TEMPERATURE FIELD AND HEAT FLUX VECTOR

Because equation (8) varies intrinsically as a function of the thermal Mach number, the temperature field should be investigated individually for $M < 1$, $M = 1$, and $M > 1$. In all cases, the temperature and its gradient are assumed to vanish at infinity.

3.1. *Subsonic case*; $M < 1$

Let us first consider a transformation on the dependent variable from $T(\xi_i)$ to $f(\xi_i)$

$$T(\xi_i) = \exp[-c\xi_1/(1-M^2)]f(\xi_i), \quad \text{for } M<1. \quad (10)$$

By substituting equation (10) into equation (8), an equation governing $f(\xi_i)$ can be found as

$$(1-M^2)f_{,11}+f_{,22}-[c^2/(1-M^2)]f = -(Q/\rho C_p\alpha)\{\delta(\xi_1)\delta(\xi_2)-(M^2/2c)\delta_{,1}(\xi_1)\delta(\xi_2)\}. \quad (11)$$

Further, by applying a successive transformation on the independent variables from ξ_i to r

$$r = \sqrt{(\xi_1^2/(1-M^2)+\xi_2^2)}, \quad \text{for } M<1 \quad (12)$$

which stretches a circle on the ξ_i-plane into an ellipse, equation (11) can be arranged in the form of

$$f_{,rr}+\frac{1}{r}f_{,r}-[c^2/(1-M^2)]f = -(Q/\rho C_p\alpha)\exp[c\xi_1/(1-M^2)]\delta(\xi_1)\delta(\xi_2) + (QM^2/2c\rho C_p\alpha)\exp[c\xi_1/(1-M^2)]\delta_{,1}(\xi_1)\delta(\xi_2) \quad (13)$$

where the independent variables ξ_i in the non-homogeneous terms of the equation are temporarily retained for later use. The solution of equation (13) consists of two parts. One is the contribution from the real heat source applied to the solid, another is resulting from the effect of finite speed of heat propagation. For both cases, the solutions can be found if the Green's function $f_g(r|r_\eta)$ satisfying the equation

$$f_{g,rr}+\frac{1}{r}f_{g,r}-[c^2/(1-M^2)]f_g = \delta(r-r_\eta) \quad (14)$$

is sought. Equation (14) is governed by a modified Bessel operator. The Green's function for this type of equation is well known [19] as

$$f_g(r|r_\eta) = \frac{1}{A} K_0[c|r - r_\eta|/(1-M^2)^{1/2}] \quad (15)$$

where

$$r_\eta = [\eta_1^2/(1-M^2) + \eta_2^2]^{1/2}$$

and the constant A comes from the Wronskian

$$I_0(z)K_0'(z) - I_0'(z)K_0(z) = \frac{A}{z} \quad (16)$$

which is obviously equal to -1. Equation (15) is the fundamental solution of equation (13). By noticing the integral properties of the Dirac delta function [1]

$$\int G[r(\xi_1,\xi_2)|r^0(\xi_1^0,\xi_2^0)]\delta(\xi_1^0)\delta(\xi_2^0)\,d\xi_1^0\,d\xi_2^0 = G[r(\xi_1,\xi_2)|r^0(0,0)]$$

$$\int G[r(\xi_1,\xi_2)|r^0(\xi_1^0,\xi_2^0)]\delta_{,1}(\xi_1^0)\delta(\xi_2^0)\,d\xi_1^0\,d\xi_2^0 = -\left\{\frac{\partial}{\partial \xi_1^0} G[r(\xi_1,\xi_2)|r^0(\xi_1^0,0)]\right\}_{\xi_1^0 \to 0} \quad (17)$$

the solution of equation (13) for $f(r)$ can be found by direct integration. This yields

$$f(r(\xi_i)) = (Q/\rho\alpha C_p)\left\{\frac{2-M^2}{2(1-M^2)} K_0[cr/(1-M^2)^{1/2}] - \frac{M^2}{2(1-M^2)} K_1[cr/(1-M^2)^{1/2}]\right\} \quad (18)$$

and the temperature $T(\xi_i)$ in the subsonic range is thus obtained from equation (10)

$$T(\xi_i)/(Q/\rho\alpha C_p) = \exp[-c\xi_1/(1-M^2)] \times \left\{\frac{2-M^2}{2(1-M^2)} K_0[cr/(1-M^2)^{1/2}] - \frac{M^2}{2(1-M^2)} K_1[cr/(1-M^2)^{1/2}]\right\}, \quad M < 1. \quad (19)$$

Note that as $M \to 0$, equation (19) is reduced to the expression in the diffusion model. The temperature gradients can be obtained immediately by taking differentiations on $T(\xi_i)$

$$T_{,1}/(Q/\rho\alpha C_p) = \exp[-c\xi_1/(1-M^2)] \times \left\{\left[\frac{M^2 c\xi_1}{2r(1-M^2)^{5/2}} - \frac{(2-M^2)c}{2(1-M^2)^2}\right] K_0[cr/(1-M^2)^{1/2}] + \left[\frac{M^2 c}{2(1-M^2)^2} - \frac{(2-M^2)c\xi_1}{2r(1-M^2)^{5/2}} + \frac{M^2\xi_1}{2r^2(1-M^2)^2}\right] \times K_1[cr/(1-M^2)^{1/2}]\right\} \quad (20)$$

$$T_{,2}/(Q/\rho\alpha C_p) = \exp[-c\xi_1/(1-M^2)] \times \left\{\left[\frac{M^2 c\xi_2}{2r(1-M^2)^{3/2}}\right] K_0[cr/(1-M^2)^{1/2}] + \left[\frac{M^2\xi_2}{2r^2(1-M^2)} - \frac{(2-M^2)c\xi_2}{2r(1-M^2)^{3/2}}\right] K_1[cr/(1-M^2)^{1/2}]\right\}. \quad (21)$$

They will be used later as we calculate the heat flux vector q.

3.2. *Supersonic case*; $M > 1$

As $M > 1$, equation (8) can be reduced to the following form:

$$\alpha[(M^2-1)T_{,11} - T_{,22}] - 2c\alpha T_{,1} = (Q/\rho C_p) \times \{\delta(\xi_1)\delta(\xi_2) - (M^2/2c)\delta_{,1}(\xi_1)\delta(\xi_2)\}. \quad (22)$$

In this case, the transformations on the dependent (from T to f) and independent (from ξ_i to r) variables are taken to be

$$T(\xi_i) = \exp[c\xi_1/(M^2-1)]f(\xi_i),$$
$$r = [\xi_1^2/(M^2-1) - \xi_2^2]^{1/2}, \quad \text{for } M > 1 \quad (23)$$

where a circle on the ξ_i-plane is stretched into a branch of hyperbola. The corresponding equation governing the function of $f(\xi_i)$ becomes

$$f_{,rr} + \frac{1}{r} f_{,r} - [c^2/(M^2-1)]f = (Q/\rho C_p\alpha) \times \exp[-c\xi_1/(M^2-1)]\delta(\xi_1)\delta(\xi_2) - (QM^2/2c\rho C_p\alpha) \times \exp[-c\xi_1/(M^2-1)]\delta_{,1}(\xi_1)\delta(\xi_2) \quad (24)$$

and the Green's function in this case is

$$f_g(r|r_\eta) = K_0[c|r - r_\eta|/(M^2-1)^{1/2}]. \quad (25)$$

By employing the same procedure as that used in the previous case, the temperature and its gradient can be obtained as

$$T(\xi_i)/(Q/\rho\alpha C_p) = -\exp[c\xi_1/(M^2-1)]\left\{\frac{2-M^2}{2(M^2-1)} K_0[cr/(M^2-1)^{1/2}] - \frac{M^2}{2(M^2-1)} K_1[cr/(M^2-1)^{1/2}]\right\}, \quad M > 1 \quad (26)$$

$$T_{,1}/(Q/\rho\alpha C_p) = \exp[c\xi_1/(M^2-1)] \times \left\{-\left[\frac{M^2 c\xi_1}{2r(M^2-1)^{5/2}} + \frac{(2-M^2)c}{2(M^2-1)^2}\right] K_0[cr/(M^2-1)^{1/2}] + \left[\frac{M^2 c}{2(M^2-1)^2} + \frac{(2-M^2)c\xi_1}{2r(M^2-1)^{5/2}} - \frac{M^2\xi_1}{2r^2(M^2-1)^2}\right] K_1[cr/(M^2-1)^{1/2}]\right\} \quad (27)$$

$$T_{,2}/(Q/\rho\alpha C_p) = \exp[c\xi_1/(M^2-1)] \times \left\{\left[\frac{M^2 c\xi_2}{2r(M^2-1)^{3/2}}\right]K_0[cr/(M^2-1)^{1/2}]+\left[\frac{M^2\xi_2}{2r^2(M^2-1)} - \frac{(2-M^2)c\xi_2}{2r(M^2-1)^{3/2}}\right]K_1[cr/(M^2-1)^{1/2}]\right\}. \quad (28)$$

Unlike the subsonic case, we first notice that the variable r defined in equation (23) restricts the applicable range of equations (26)–(28) to the domain satisfying the condition

$$\xi_1^2/(M^2-1) > \xi_2^2. \quad (29)$$

The physical interpretation of equation (29) can be nicely made if we represent the condition in terms of the polar coordinate (R, θ) centered at the heat source, as shown in Fig. 4. The equivalent expression of equation (29) is then

$$|\tan\theta| < [1/(M^2-1)]^{1/2} \quad (30)$$

or in a more familiar form

$$0 < \theta < \sin^{-1}(1/M);$$

for a heat source moving to the left (31)

$$\pi - \sin^{-1}(1/M) < \theta < \pi;$$

for a heat source moving to the right (32)

where the domain of θ in the third and the fourth quadrant is omitted due to symmetry of the problem. Equation (32) is the one assumed in this analysis. The temperature field outside this domain stays undisturbed as the heat source moves to the right with a supersonic speed. The oblique thermal shock wave coincides with the asymptotes of the hyperbola which inclines an angle $\sin^{-1}(1/M)$ with the negative x_1- or ξ_1-axis. Obviously, the heat affected zone represented by equation (32) becomes narrower as the thermal Mach number increases. This is a similar situation to that which occurred in supersonic aerodynamics.

3.3. *Transonic case*; $M = 1$

As the heat source moves at the same speed as the heat propagation speed in the solid, equation (8) is degenerated into a parabolic equation

$$T_{,22}+2cT_{,1} = -(Q/\rho\alpha C_p)\{\delta(\xi_1)\delta(\xi_2) - (M^2/2c)\delta_{,1}(\xi_1)\delta(\xi_2)\}. \quad (33)$$

The Green's function for this equation is standard and can be expressed as [20]

$$T_g(\xi_i|\xi_i^0) = [-1/(8\pi c)^{1/2}] \times \exp[-c(\xi_2-\xi_2^0)^2/2|\xi_1-\xi_1^0|]/[|\xi_1-\xi_1^0|]^{1/2} \quad (34)$$

where ξ_i^0, $i = 1, 2$ denote the location of the heat source. The temperature distribution and its derivatives in this case are

$$T/(Q/\rho\alpha C_p) = [1/(8\pi c)^{1/2}]\exp(c\xi_2^2/2\xi_1) \times [1/(-\xi_1)^{1/2}+(c\xi_2^2+\xi_1)/4c(-\xi_1)^{5/2}] \quad (35)$$

$$T_{,1}/(Q/\rho\alpha C_p) = [1/(8\pi c)^{1/2}]\exp(c\xi_2^2/2\xi_1) \times [1/8c(-\xi_1)^{7/2}]\{c(4\xi_1^2+5\xi_2^2)+3\xi_1 + (c\xi_2^2/\xi_1)[c(4\xi_1^2+\xi_2^2)+\xi_1]\}$$

$$T_{,2}/(Q/\rho\alpha C_p) = [-1/(8\pi c)^{1/2}]\exp(c\xi_2^2/2\xi_1) \times [\xi_2/4(\xi_1)^{7/2}][c(4\xi_1^2+\xi_2^2)+3\xi_1]. \quad (36)$$

A singularity at $\xi_1 = 0$ exists in the solutions given by equations (34)–(36) the applicable range of which is restricted to the domain of $\xi_1 < 0$. This is a consistent result with that obtained in the supersonic case with $M \to 1$. The heat affected zone in this case is from 90 to 180 deg, and a normal shock wave is persisting to the heat source as it moves to the right.

The heat flux vector q with components q_1 and q_2 in the ξ_i coordinate system can be obtained by integrating equation (9). The integral representations of the results are

$$\{q_1, q_2\} = k\int_\infty^0 e^{-\zeta}\left\{T_{,1}\left(\xi_1+\frac{M^2}{2c}\zeta, \xi_2\right), T_{,2}\left(\xi_1+\frac{M^2}{2c}\zeta, \xi_2\right)\right\}d\zeta \quad (37)$$

where q_1 and q_2 are assumed to vanish at a distance far away from the heat source. With the temperature gradients obtained in various ranges of M, namely equations (20) and (21) for $M < 1$, equations (27) and (28) for $M > 1$, and equation (36) for $M = 1$, equation (37) can be integrated for q_1 and q_2 in the subsonic, supersonic, and transonic ranges. Although analytical forms of the solutions for q's are difficult to obtain due to complexity of the integrand, the integral involved in equation (37) obviously exists since all the functions vanish at a large value of ζ.

4. NUMERICAL EXAMPLES

We will first present the temperature distributions along the circumference of a circle centered at the heat source. The radius of the circle is taken to be 10^{-4} m which is approximately the lower bound of continuum mechanics beyond which microscopic lattice structure of the medium can be neglected in the formulation. The material behavioral parameters are assumed to be those for aluminum alloys

$$\rho = 2.72\times10^3\ \text{kg m}^{-3},\quad k = 220\ \text{W m}^{-1}\text{K}^{-1}, \quad C_p = 0.895\ \text{kJ kg}^{-1}\text{K}^{-1} \quad (38)$$

and the speed of heat propagation C in the solid is taken as 20 m s^{-1}. Under the present formulation, it should be noticed that the parametric value of C is absorbed in the thermal Mach number M in the field solutions. It only serves as a reference value for esti-

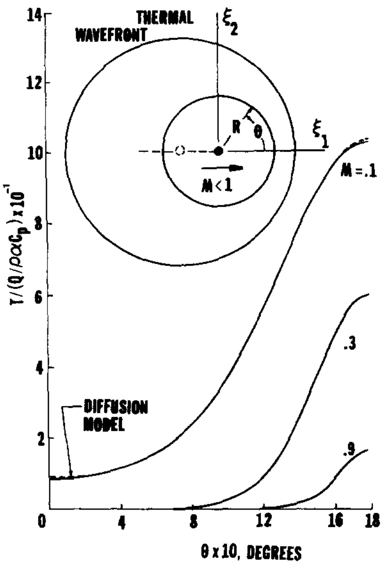

FIG. 2. Temperature distribution along the circumference of the continuum circle centered at the heat source—subsonic case, $M < 1$.

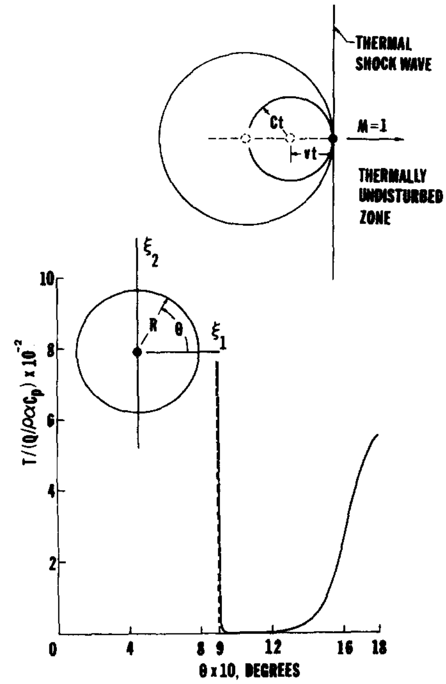

FIG. 3. Temperature distribution and shock wave formation at transonic stage, $M = 1$.

mating the relative speed v of the moving heat source through the parameter c.

Figure 2 shows the temperature distribution represented by equation (19) for the subsonic case with $M < 1$. In comparing the curves with different values of M, we observe that the temperature level decreases as the value of M increases. At a fixed value of M, we notice that the temperature increases slowly as θ is small. While as the trailing edge of the heat source is approaching, it tends to increase with a much higher rate and for all the cases under consideration, the temperature reaches its maximum at $\theta = 180$ deg. Also, we notice that as the heat source approaches the transonic stage ($M \to 1$), the thermally undisturbed zone is gradually formed. This is reflected by the presence of a domain of θ (starting from $M = 0.3$ approximately) within which only a minor temperature increase is observed. Such a domain increases as the value of M increases. In tribology with high-speed friction load, the thermal Mach number can reach as high as 0.4–0.6 [21]. According to the temperature distributions shown in Fig. 2, significant deviations between the thermal diffusion and the thermal wave models are observed for M being in this range.

As $M = 1$, the temperature distribution represented by equation (35) is shown in Fig. 3. A normal shock wave is formed in the physical domain. The thermal field in the domain of $0 < \theta < 90$ deg stays undisturbed as the heat sources move ahead. We observe that except for the singularity existing at the thermal shock wave, the temperature level in the heat affected zone continuously decreases in comparison with those in the subsonic range. The temperature distribution represented by equation (26) for the supersonic case is displayed in Fig. 4. Distinct shock waves are formed at $\theta = 150$, 168.46, and 172.82 deg for $M = 2$, 5, and 8, respectively. The physical domain for the thermally undisturbed zone dramatically increases from 90 to 150 deg as the thermal Mach number increases from 1 to 2, but the rate of increasing gradually tapers off as the value of M further increases. By comparing these distributions, we notice that the temperature

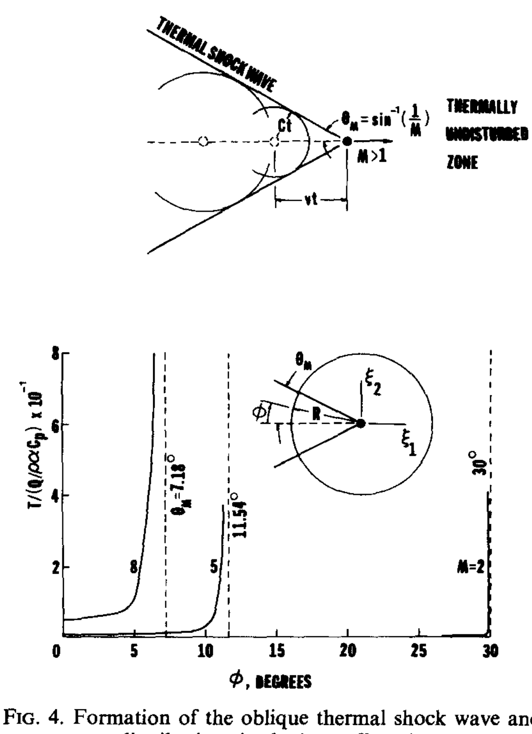

FIG. 4. Formation of the oblique thermal shock wave and temperature distributions in the heat affected zone—supersonic case, $M > 1$.

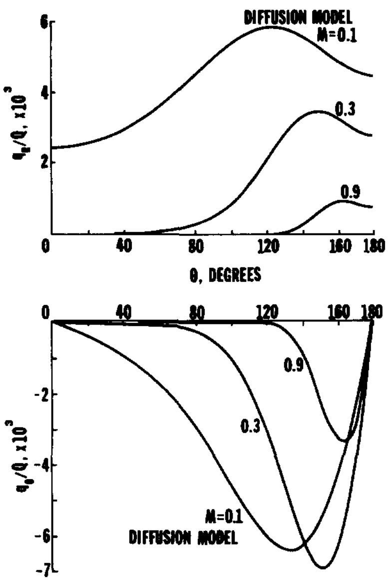

FIG. 5. Distributions of the heat flux components q_R and q_θ in the subsonic range.

level continuously decreases as the value of M increases to 2. It then increases with the thermal Mach number as the value of M further increases.

The distributions of the heat flux components q_R and q_θ in various ranges of M are shown in Figs. 5–8. They are related to the components of q_1 and q_2 by

$$q_R = q_1 \cos\theta + q_2 \sin\theta \quad \text{and}$$

$$q_\theta = -q_1 \sin\theta + q_2 \cos\theta. \quad (39)$$

The lower bound ∞ appearing in the improper integral of equation (37) is approached by an integer parameter B such that the difference between the results of two successive integrations, with B and $B-1$ as the lower bounds, respectively, is less than 1% of the current value. For all the values of M ranging from 0.1 to 5 considered in the present study, the values for B to achieve such an accuracy are less than 10 by employing Simpson's rule. All the physical trends existing in the temperature field are preserved in the distributions of q_R and q_θ. Due to presence of the thermal shock waves at the transonic stage, q_θ switches to be positive in the vicinity of the shock wave singularity, as shown by Fig. 6. In the supersonic ranges with $M > 1$, as shown by Fig. 7, q_θ stays positive in

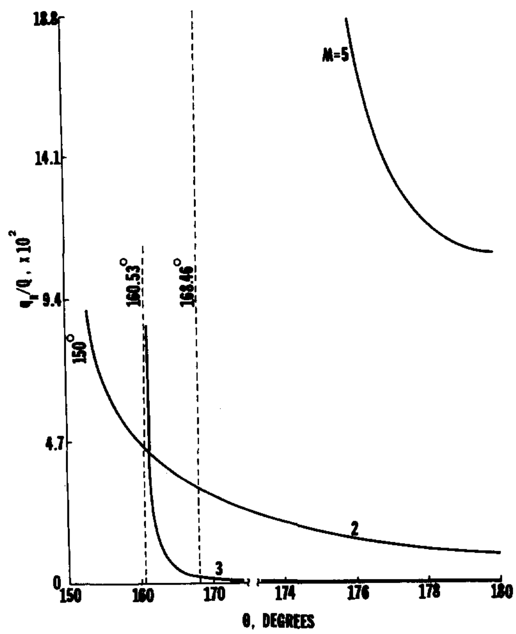

FIG. 7. Distributions of the heat flux components q_R in the supersonic range.

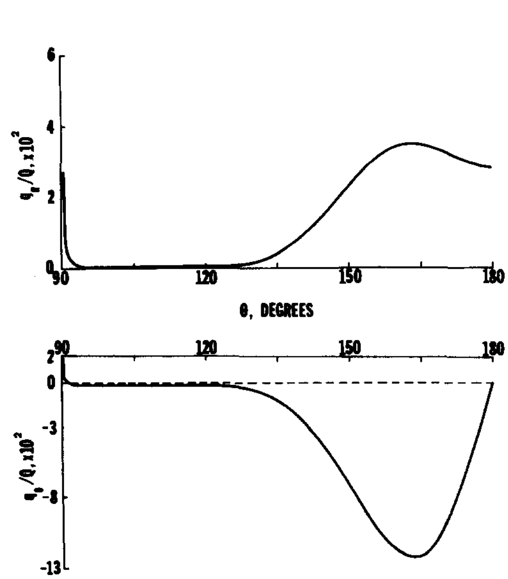

FIG. 6. Distributions of the heat flux components q_R and q_θ at the transonic stage.

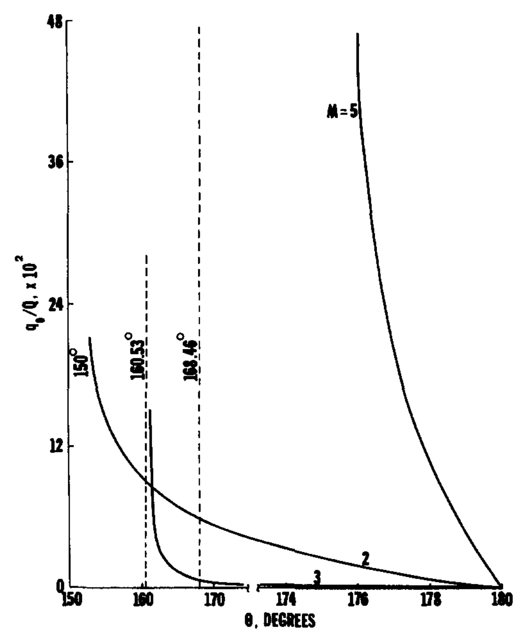

FIG. 8. Distributions of the heat flux components q_θ in the supersonic range.

the entire domain of the heat affected zone due to the strong influence of the shock wave singularity. Also, the swinging phenomenon can be clearly observed in Figs. 7 and 8. The value of M beyond which the heat flux increases with the thermal Mach number is approximately equal to 3.

5. DISCUSSION

Several salient features existing in the hyperbolic theory of heat conduction with a moving heat source have been investigated. It includes, for example, the formation of the thermal shock wave and the thermally undisturbed zone in the physical domain as the speed of the moving heat source exceeds that of the heat propagation in the solid ($M \geqslant 1$). Most importantly, a swinging phenomenon seems to exist for the variations of temperature level around the heat source vs the thermal Mach number. Referring to Figs. 2 and 4, we notice that in the subsonic range with $M < 1$, the temperature level in the neighborhood of the moving heat source decreases as the thermal Mach number increases. While entering the supersonic range with $M > 2$ approximately, the temperature level in the heat affected zone starts to increase with the thermal Mach number. The physical reason for such a variation in the subsonic range is understandable. For $M < 1$, which means either the speed of the moving heat source is low or the speed of heat propagation in the solid is high, the material continua in the vicinity of the heat source do not have sufficient time to respond to the appropriate temperature level before the heat source moves ahead, and consequently the temperature level nearby decreases as the thermal Mach number increases. In the supersonic range with $M \geqslant 1$, however, the thermal energy tends to accumulate along a preferred direction at $\theta_M = \sin^{-1}(1/M)$ which results in the formation of the thermal shock and the temperature nearby approaches infinity. As the thermal Mach number increases, referring to equation (32), the physical domain of the heat affected zone bounded by the thermal shock waves reduces and the effects of the temperature singularity at the thermal shock wave essentially promotes the temperature level in the heat affected zone. For the present problem, such an effect is activated for the thermal Mach number being in the threshold of 2 and thereafter.

An immediate application of the present analysis is the prediction of the failure modes around the moving heat source [22]. As a result of the thermal Mach number appearing in the energy equation for the thermal wave model, the regular equation governing the thermoelastic displacement potential Φ is modified to the following form:

$$M^2\Phi_{,11} - 2c\Phi_{,1} = \varepsilon\beta[\Phi_0(\xi_i) + T] \qquad (40)$$

with ε being the coefficient of thermal expansion, β a function of the Poisson's ratio, Φ_0 a function of ξ_i resulting from the apparent heat source, and T the temperature in the subsonic, transonic and supersonic ranges, respectively. The characteristic of this equation is identical to that of equation (9) governing the heat flux q. But the resulting thermal stresses involving various orders of spatial differentiations on Φ are much more complicated than the solution for q represented by equation (37). In the case that M approaches zero, equation (40) is reduced to the ordinary representation employing the thermal diffusion model. The modified term is led by M^2 which appears as the highest order differential in the modified equation. Mathematically, a dramatic change in the characteristics of the solution for Φ, and hence for the thermal stress field, is expected. The limiting case with M approaching infinity ($M \to \infty$) is interesting to note. According to equation (32), the heat affected zone in this case is confined to a narrow band at $\theta = 0$ and the thermal energy tends to be accumulated at the trailing edge of the moving heat source. The thermal stress due to such a great amount of energy accumulation also approaches a large value and the resulting material failure consequently occurs at the trailing edge of the moving heat source. This is an expected result based on our experience but it cannot be depicted by the thermal diffusion model. According to the classical theory of thermal diffusion, the temperature around a moving heat source continuously decreases as the speed of the moving heat source increases. Under the limiting case with $M \to \infty$, the temperature around the moving heat source reduces to the reference value (room temperature) and no material failure could be observed except underneath the heat source.

Apparently, consideration of the thermal wave model for the problems with singularities is necessary. This includes the presence of a discontinuous thermal loading imposed on the medium, a macrocrack existing in the solid, or the interfacial area between dissimilar materials. The rate change of temperature or temperature gradient in these regions is usually high and the deviations between the thermal diffusion and the thermal wave models will be significant.

REFERENCES

1. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, Vol. 1, pp. 837–839 and 865. MacGraw-Hill, New York (1953).
2. M. Chester, Second sound in solids, *Phys. Rev.* **131,** 2013–2015 (1963).
3. H. D. Weymann, Finite speed of propagation in heat conduction, diffusion, and viscous shear motion, *Am. J. Phys.* **35,** 488–496 (1967).
4. Y. Taitel, On the parabolic, hyperbolic and discrete formulation of the heat conduction equation, *Int. J. Heat Mass Transfer* **15,** 369–371 (1972).
5. M. E. Gurtin and A. C. Pipkin, A general theory of heat conduction with finite wave speeds, *Arch. Rational Mech. Analysis* **31,** 113–126 (1968).
6. V. A. Bubnov, Kinetic theory of heat conduction, *Int. J. Heat Mass Transfer* **19,** 175 (1976).
7. B. M. Berkovsky and V. G. Bashtovoi, The finite speed

of heat propagation from the viewpoint of the kinetic theory, *Int. J. Heat Mass Transfer* **20**, 621–626 (1977).
8. N. G. Van Kampen, A model for relativistic heat transport, *Physica* **46**, 315–332 (1970).
9. D. C. Kelly, Diffusion: a relativistic appraisal, *Am. J. Phys.* **36**, 585–591 (1968).
10. J. I. Frankel, B. Vick and M. N. Özisik, Flux formulation of hyperbolic heat conduction, *J. Appl. Phys.* **58**(9), 3340–3345 (1985).
11. K. J. Baumeister and T. D. Hamill, Hyperbolic heat-conduction equation—a solution for the semi-infinite body problem, *ASME J. Heat Transfer* **91**, 543–548 (1969).
12. D. C. Wiggert, Analysis of early-time transient heat conduction by method of characteristics, *ASME J. Heat Transfer* **99**, 35–40 (1977).
13. M. N. Özisik and B. Vick, Propagation and reflection of thermal waves in a finite medium, *Int. J. Heat Mass Transfer* **27**, 1845–1853 (1984).
14. B. Vick and M. N. Özisik, Growth and decay of a thermal pulse predicted by the hyperbolic heat conduction equation, *ASME J. Heat Transfer* **105**, 902–907 (1983).
15. D. Y. Tzou, On the thermal shock waves induced by a moving heat source, *ASME J. Heat Transfer* **111**, 232–238 (1989).
16. D. Y. Tzou, Intensification of externally applied magnetic field around a crack in layered composite, *J. Theoretical Appl. Fracture Mech.* **4**, 191–199 (1985).
17. D. Y. Tzou and G. S. Sih, Discussion on the criteria for brittle fracture in biaxial tension, *J. Engng Fracture Mech.* **21**, 977–981 (1985).
18. A. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 1, pp. 304–308. Ronald Press, New York (1954).
19. G. Arfken, *Mathematical Methods for Physicists* (2nd Edn), p. 763. Academic Press, New York (1970).
20. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, p. 255. Oxford University Press, London (1959).
21. F. D. Ju and J. C. Liu, Effect of Peclet number in thermomechanical cracking due to high-speed friction load, *ASME J. Tribology* **21**, 1–5 (1987).
22. D. Y. Tzou, Thermoelastic fracture induced by the thermal shock waves around a moving heat source, *ASME 1989 National Heat Transfer Conf.*, HTD-113, *Heat Transfer in Manufacturing and Material Processing*, pp. 11–17 (1989).

FORMATION D'ONDE DE CHOC AUTOUR D'UNE SOURCE DE CHALEUR MOBILE DANS UN SOLIDE AVEC VITESSE FINIE DE PROPAGATION DE LA CHALEUR

Résumé—On étudie analytiquement le champ thermique autour d'une source de chaleur mobile dans un solide avec vitesse finie de propagation de la chaleur. Un nombre de Mach thermique M défini comme le rapport entre la vitesse de la source thermique et celle de la propagation dans le solide est introduit dans l'analyse. L'équation de l'énergie est elliptique, parabolique ou hyperbolique respectivement dans le domaine subsonique ($M < 1$), transonique ($M = 1$) et supersonique ($M > 1$). L'onde de choc thermique existe dans le domaine physique lorsque la vitesse de la source de chaleur est égale ou supérieure à celle de la propagation de la chaleur, et l'angle du choc thermique est égal à $\sin^{-1}(1/M)$ pour $M \geqslant 1$. Pour les exemples numériques traités, l'évolution de température et de flux thermique dans les zones affectées par la chaleur est présentée en fonction du nombre de Mach thermique et on discute un phénomène d'agitation du champ thermique en transition.

DIE BILDUNG EINER THERMISCHEN STOSSWELLE UM EINE IN EINEM FESTKÖRPER BEWEGTE WÄRMEQUELLE BEI ENDLICHER AUSBREITUNGSGESCHWINDIGKEIT DER WÄRME

Zusammenfassung—Im Vordergrund der Arbeit steht die analytische Untersuchung des Temperaturfeldes um eine bewegte Wärmequelle in einem Festkörper bei endlicher Ausbreitungsgeschwindigkeit der Wärme. Dabei wird eine thermische Mach-Zahl (M) definiert, die sich aus dem Verhältnis der Bewegungsgeschwindigkeit der Wärmequelle und der Ausbreitungsgeschwindigkeit der Wärme ergibt. Die Energiegleichung ist im Unterschallbereich ($M < 1$) elliptisch, bei Schallgeschwindigkeit ($M = 1$) parabolisch und im Überschallbereich ($M > 1$) hyperbolisch. Ist die Geschwindigkeit der Wärmequelle größer oder gleich der Ausbreitungsgeschwindigkeit der freigesetzten Wärme, so zeigt sich eine thermische Stoßwelle, deren Winkel analytisch mit arcsin $(1/M)$ berechnet werden kann. Anhand von numerischen Beispielen wird die Entwicklung der Temperatur- und Wärmestromverteilung in der von der Wärmequelle beeinflußten Zone als Funktion der thermischen Mach-Zahl dargestellt. Das Schwingverhalten des transienten thermischen Feldes wird erörtert.

ОБРАЗОВАНИЕ УДАРНЫХ ВОЛН ВОКРУГ ДВИЖУЩЕГОСЯ ИСТОЧНИКА ТЕПЛА В ТВЕРДОМ ТЕЛЕ С КОНЕЧНОЙ СКОРОСТЬЮ РАСПРОСТРАНЕНИЯ ТЕПЛА

Аннотация—Аналитически исследуется тепловое поле около движущегося источника тепла в твердом теле с конечной скоростью распространения тепла. В анализе используется тепловое число Маха M, определяемое как отношение скоростей перемещения теплового источника и распространения тепла в твердом теле. Найдено, что полученное уравнение сохранения энергии является соответственно эллиптическим, параболическим и гиперболическим в диапазонах дозвуковой ($M < 1$), околозвуковой ($M = 1$) и сверхзвуковой ($M > 1$) скоростей. Показано, что тепловая удрная волна существует в физической опласти, т.к. скорость движущегося источника тепла равна или превышает скорость распространения тепла, также получено аналитическое выражение угла наклона ударной волной $\sin^{-1}(1/M)$ при $M \geqslant 1$. В численных примерах развитие распределений температуры и теплового потока в зоне, подверженной тепловому воздействию, представлено как функция теплового числа Маха; обсуждается явление качания для случая теплового поля в переходном состоянии.